Manipulation of Majorana fermions via single charge control

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Outline

1D topological superconductor and Majorana bound states

Tunnel characteristic of chain of Majorana bound states

Spectroscopy with a Coulomb island coupled to Majorana state

Non-abelian manipulation via single electron control

Transfer between topological and spin qubit systems
Topological superconductor

Semiconductor with strong S0

S-wave superconductor

\[ H_0 = \frac{p_x^2}{2m} + \alpha (\mathbf{E} \times \mathbf{p}_x) \cdot \sigma + \frac{1}{2} g \mu_B \mathbf{B} \cdot \sigma \]

\( B = 0 \)

\( B \neq 0 \)

\( m = \alpha = 1 \)

Can now couple to s-wave superconductor
Triplet superconductor by Zeeman & SO

Pairing in semiconductor induced by proximity effect:

\[ \Delta = 0 \]
\[ \Delta = 0.5B \]
\[ \Delta = B \]
\[ \Delta = 1.5B \]

Effective Hamiltonian for the lowest mode \( \mu = 0 \):

\[
H_{\text{eff}} = \frac{i\alpha}{2} \int dx \{ \eta_1(x) \partial_x \eta_1(x) - \eta_2(x) \partial_x \eta_2(x) \} + i \int dx \{ i(B_\perp(x) - \Delta(x))\eta_1(x)\eta_2(x) \} 
\]

Spin up/down Majorana modes:

\[ \eta_1(x) = \Psi_\uparrow^\dagger(x) + \Psi_\downarrow^\dagger(x) \]
\[ \eta_2(x) = \Psi_\uparrow^\dagger(x) + \Psi_\downarrow(x) \]
Majorana end bound states

Topological superconductor

Normal lead

\[ \gamma_1 = \int \! dx \, f_L(x) \left( \Psi_{\uparrow y}(x) + \Psi_{\uparrow y}^\dagger(x) \right) \] : Spin up in \( y \)

\[ \gamma_2 = \int \! dx \, f_R(x) \left( \Psi_{\downarrow y}(x) + \Psi_{\downarrow y}^\dagger(x) \right) \] : Spin down in \( y \)

\[ H_T = \sum_k \left( v_{1,k} c_{k\uparrow y} - v_{1,k}^* c_{k\uparrow y}^\dagger \right) \gamma_1 \]
Tunneling into disordered chain of Majorana bound states

\[ H_S = \frac{i}{2} \sum_{ij} t_{ij} \gamma_i \gamma_j \]

\[ \mu_c = \pm \sqrt{B^2 - \Delta^2} \]

(Disorder correlation length > Majorana localization length)
General current formula

\[
\frac{dI}{dV} = \frac{2e^2}{h} \int d\omega \Gamma \text{Im}[G_{11}^R(eV)] \left[ \frac{df(\omega - eV)}{d\omega} \right]
\]

\[
G^R(\omega) = 2[\omega - 2it + i2\Gamma]^{-1}
\]

PRB, 180516R (2010)
Short chain

Andreev channel: resonant

Electrons  MBS  Holes

\[ \mu + \Delta \]
\[ \mu \]
\[ \mu - \Delta \]
Long random chain

Example with “weak” link:
What is a "weak" link?

Remember:

$$\frac{dI}{dV} = \frac{2e^2}{h} \int d\omega \Gamma \text{Im}[G^R_{11}(eV)] \left[ \frac{df(\omega - eV)}{d\omega} \right]$$

$$t_{\text{weak}} = t_{n,n+1}$$

Width of resonances due to the chain after weak link:

$$\Gamma_{\text{weak}} \propto \frac{t_{\text{weak}}^2}{\langle t \rangle \Gamma}$$

Note visible if: $$k_B T \gtrsim \Gamma_{\text{weak}}$$
Spectroscopy of Majorana bound states using quantum dot

Tunneling limit

\[ \Gamma \ll U, \lambda, k_BT \]

Solve QD-TS exactly and use master equation - include relaxation of parity
Eigenstates of QD-TS

Even subspace, 4 states:
|00⟩, |σ1⟩, |20⟩

Odd subspace, 4 states:
|σ0⟩, |01⟩, |21⟩

Mixed by tunneling

Solve rate equations!
Coulomb blockade "diamond"

\[ \lambda = T, \xi = 0 \]

\[ \lambda = 5T, \xi = 0 \]

\[ \sigma = \uparrow \]

\[ \sigma = \downarrow \]

Bias voltage

\[ \alpha V_g \] [T]

Note:
Blue = NDR

Zero bias peak

MBS
Finite coupling between Majorana states – inelastic cotunneling - relaxation

\[ \lambda = T, \xi = 10T \]

\[ \Lambda = 4\Gamma \lambda^2 / \xi^2 \]

Non-eq. Cotun.
Non-abelian manipulations using single charge control

By changing the charge on a dot by one electron:

\[ P_{12} : |i\rangle \mapsto (|v_1\gamma_1 + |v_2\gamma_2|i\rangle \]

Compare to braiding:

\[ B_{12} : |i\rangle \mapsto \frac{1}{\sqrt{2}} (1 + \gamma_1\gamma_2)|i\rangle \]
Manipulation of the Majorana system by single electron addition/removal

Projection to zero mode and one spin direction in dot:

\[ H_1 = \varepsilon c_{1_y}^{\dagger} c_{1_y} + \left( v_1^{*} c_{1_y}^{\dagger} - v_1 c_{1_y} \right) \gamma_1 \]

M-fermion:
\[ d = (\gamma_1 + i\gamma_2)/2 \]
\[ \gamma_1 = d + d^{\dagger} \]

Basis states:
Total even: \( \{ |0\rangle_D |0\rangle_{M12}, |1\rangle_D |1\rangle_{M12} \} \)
Total odd: \( \{ |0\rangle_D |1\rangle_{M12}, |1\rangle_D |0\rangle_{M12} \} \)

\[ H_{1,\text{even/odd}} = \begin{pmatrix} 0 & v_1 \\ v_1^{*} & \varepsilon \end{pmatrix} \]

"Protected"
More Majorana states and more dots

Allows a finite number of operations of the form:

$$\gamma_1 \cdots \gamma_m |i\rangle_M$$
Coupling to two Majorana bound states

M1, M2 and D1:

\[ H_{12} = \varepsilon c_{\uparrow y}^{\dagger} c_{\uparrow y} + \left( v_1^* c_{\downarrow y}^{\dagger} - v_1 c_{\downarrow y} \right) \gamma_1 + \left( v_2^* c_{\downarrow y}^{\dagger} - v_2 c_{\downarrow y} \right) \gamma_2 \]

But now even/odd not degenerated

\[ H_{12, \text{even/odd}} = \begin{pmatrix} 0 & v_{\text{even/odd}} \\ v_{\text{even/odd}}^* & \varepsilon \end{pmatrix} \]

\[ v_{\text{even/odd}} = v_1 \mp i v_2 \]

(degenerated only if \( v_1 \) and \( v_2 \) are real)
In degeneracy point: \[ 2\text{Arg}(v_1/v_2) = 0 \]

\[
H_{12} = \varepsilon \hat{c}^\dagger \hat{c} + \nu (\hat{c}^\dagger - \hat{c}) \gamma_{12}
\]

\[
\gamma_{12} = \frac{1}{\sqrt{|v_1|^2 + |v_2|^2}} (|v_1|\gamma_1 + |v_2|\gamma_2)
\]

\[
P_{12} : \quad |i\rangle \mapsto \gamma_{12} |i\rangle
\]

Requires:
- Constant tunneling amplitudes
- Constant flux

No dependence on timing
Compare to braiding

\[ \gamma_{12} = \frac{1}{\sqrt{|v_1|^2 + |v_2|^2}} (|v_1| \gamma_1 + |v_2| \gamma_2) \]

With

\[ v_1 = v_2 \]

\[ F_i = \frac{1}{\sqrt{2}} (\gamma_i + \gamma_{j+1}) \]

\[ B_i = F_i \gamma_i = \gamma_{i+1} F_i \]

"Tunnel braid" can mimic real space braiding

Ivanov, PRL 2001
General $v_1$ and $v_2$

Two-level system

$\sigma_x = \gamma_1, \sigma_y = \gamma_2$

$\sigma_z = -i\gamma_1\gamma_2$

Rotation around a line in the x-y plane:

$P_{12} : |i\rangle \mapsto (|v_1\rangle \sigma_x + |v_2\rangle \sigma_y) |i\rangle$

Rotation around the z-axis:

$P_{12}P'_{12}$

(Braiding is restricted to $\pi/2$ rotations)
Demonstration of non-Abelian operations

Two fermions:

\[ d_1 = \left( \gamma_1 + i \gamma_2 \right)/2 \]
\[ d_2 = \left( \gamma_3 + i \gamma_4 \right)/2 \]

Basis: \[ |n_1 n_2 \rangle \]

\[ F_i = \frac{1}{\sqrt{2}} \left( \gamma_i + \gamma_{j+1} \right) \]

Initialize: \[ |00 \rangle \]

To initialize and read out: detune away from degeneracy point
Coupled spin qubits and Majorana qubits

**Goal:** transfer quantum information from spin qubits to topological qubits, without being prone to charge noise

\[
H = H_D + \gamma_1 \left( \lambda_1 d_\uparrow - \lambda_1^* d_\uparrow^\dagger \right) + \gamma_2 \left( \lambda_2 d_\downarrow - \lambda_2^* d_\downarrow^\dagger \right)
\]
Filling or emptying the dot: entangles

Sweep from empty to full dot:

\[ P_F = \frac{1}{\lambda} \left( -\lambda_1^* \gamma_1 d_\uparrow^\dagger - \lambda_2^* \gamma_2 d_\downarrow^\dagger \right) \]

Sweep from full to empty dot:

\[ P_E = N \left( \lambda_1 \gamma_1 d_\uparrow + \lambda_2 \gamma_2 d_\downarrow \right) \]

\( N \): normalization factor

Possibilities:
- Transferring between spin & topological qubits
- Generating long-distance entanglement

Read more: arXiv:1107.5703
- Majorana bound states give clear spectroscopic features, both with and without a quantum dot.

- A Coulomb blockade setup is also sensitive to the parity relaxation.

- Single charge control allows one-qubit rotations.

- Non-Abelian manipulation using quantum dots coupled to two Majorana bound states.

\[ F_i = \frac{1}{\sqrt{2}} (\gamma_i + \gamma_{i+1}) \]

- Spin selective tunneling allows transfer of quantum information between spin and topological qubits, without charge coupling.