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Quantum communication with quantum dot spins

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Single electron spins in quantum dots are attractive for quantum communication because of their expected long coherence times. We propose a method to create entanglement between two remote spins based on the coincident detection of two photons emitted by the dots. Local nodes of two or more qubits can be realized using the dipole-dipole interaction between trions in neighboring dots and spectral addressing, allowing the realization of a quantum repeater. We have performed a detailed feasibility study of our proposal based on tight-binding calculations of quantum dot properties.

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Implementations of quantum information protocols in the solid state are of interest because they may eventually be more scalable than other approaches. Individual electron spins in quantum dots[1] are a promising system for quantum computing and quantum communication due to their expected long coherence times. Spin relaxation times as long as 20 ms have been observed at 4 T and much longer times predicted for lower magnetic fields.[2] There are theoretical predictions that in the absence of nuclear spins the decoherence time of the spins might approach their relaxation time.[3] Nuclear spins can be eliminated completely, e.g., by using isotopically purified II-VI materials, since Zn, Cd, Mg, Se, and Te all have dominant isotopes without nuclear spins.

For quantum commmunication it is important to be able to create entanglement between remote spins.[4,5] The recent proposal of Ref. 4 relies on achieving a large Faraday rotation for a single photon due to the quantum dot spin. It requires very high-finesse microcavities that are switchable in a picosecond. The proposal of Ref. 5 relies on the detection of a single photon that could have been emitted by either of two remote sources.[6] This approach is attractive because it does not require a finely controlled strong spin-photon interaction. A practical drawback of the scheme of Ref. 5 is the requirement of phase stability over the whole distance. Reference 7 proposed a scheme that creates entanglement between two remote emitters via the detection of two photons, which eliminates this stability requirement, while keeping the advantages of an emission-based scheme. In the present work we demonstrate, first, how to realize a similar scheme for quantum dot spins. Second, we show that it is possible to realize local nodes of two or more spins using dipole-dipole interactions and spectral addressing. Such nodes allow the realization of quantum repeater protocols.[5,8] We have investigated the feasibility of our proposal in detail, including numerical calculations of the electronic properties of quantum dots using tight binding methods.

Our scheme applies to flat quantum dots, such as typical strain-induced quantum dots or dots in heterostructured nanowires.[9] This implies that the lowest-energy hole states will have predominantly "heavy-hole" character, and will be well separated from predominantly "light-hole" states. The dots can be charged with single electrons via tunneling controlled by an electric field as in Ref. 10. A magnetic field is applied in the growth direction. The qubit states are the two spin states corresponding to the lowest electron level in the dot, denoted by $|1/2\rangle$ and $|-1/2\rangle$. We use transitions between the qubit states and the two lowest-energy trion states, which have angular momentum 3/2 and −3/2, see Fig. 1. A trion consists of the electron that is trapped in the dot plus an exciton (i.e., an electron-hole pair created by the incoming light). The two electrons form a spin singlet, the angular momentum of the trion is therefore determined by that of the hole, which is ±3/2. Note that a Lambda system like in Ref. 7 could be realized by applying a transverse magnetic field. However, we prefer the configuration with the field in growth direction because it makes it much easier to realize qubit measurements and two-qubit gates.

The protocol for entanglement creation starts by creating a superposition state $\frac{1}{\sqrt{2}}(|1/2\rangle+|-1/2\rangle)$ of the spin via a single-qubit rotation as described below. Then one applies simultaneous π pulses to both the $|1/2\rangle \rightarrow |3/2\rangle_T$ and the $|-1/2\rangle \rightarrow |-3/2\rangle_T$ transitions, creating the state $\frac{1}{\sqrt{2}}(|3/2\rangle_T + |-3/2\rangle_T)$. This state will decay under photon emission, creating an entangled spin-photon state $\frac{1}{\sqrt{2}}(|1/2\rangle|\sigma_+\rangle$

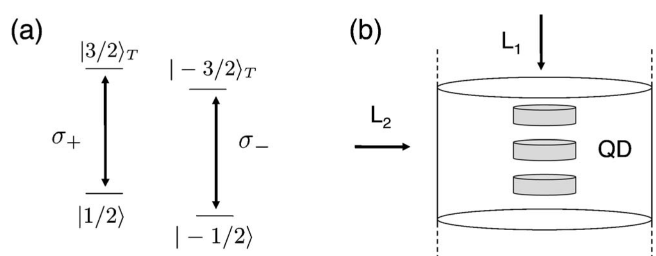

FIG. 1. (a) Level scheme underlying entanglement creation, qubit measurement and two-qubit gates. The $|1/2\rangle$ electron state is coupled to the $|3/2\rangle_T$ trion state by σ_+ radiation propagating along the growth direction, while the $|-1/2\rangle$ electron state is coupled to the $|-3/2\rangle_T$ trion by σ_- radiation. The other transitions, which have $|\Delta J|=2$, are strongly suppressed, cf. below. The photon energies are $E_{\sigma_\pm}=E_T \pm E_Z$, where E_T is the trion energy in zero field and $E_Z = g_X \mu_B B$ is the Zeeman energy, with g_X the g factor of the exciton. (b) Outline of a single node. A stack of quantum dots (QD) is embedded in a waveguide. L_1 is a control beam addressing the transitions in (a), a perpendicular beam L_2 is required for one-qubit gates. The waveguide ensures efficient collection of the emitted photons.

$+|-1/2\rangle|\sigma_-\rangle)$, where the photon states $|\sigma_+\rangle$ and $|\sigma_-\rangle$ differ not only in polarization but also in energy. Such spin-photon entangled states are created for two remote quantum dots A and B, which have been carefully tuned such that $E^A_{\sigma_+}=E^B_{\sigma_+}$ and $E^A_{\sigma_-}=E^B_{\sigma_-}$. In this case the photons from A and B will be indistinguishable for each polarization, which makes it possible to perform a partial Bell state analysis on them using only linear optical elements.[7,11] The method is based on the fact that only the antisymmetric state $|\sigma_+\rangle_A|\sigma_-\rangle_B - |\sigma_-\rangle_A|\sigma_+\rangle_B$ leads to coincidences between the two output ports if both photons are combined on a beam splitter. The required two-photon interference occurs even if the photon energies corresponding to the two polarizations are different (in contrast to what is implied in Ref. 7). The emitted photons can be collected efficiently and guided to the location of their joint measurement using waveguides and optical fibers; e.g., Ref. 12 obtained a coupling coefficient of 95% for a monochromatic emitter inside a single-mode waveguide. The Bell measurement of the photons projects the two remote spins into an entangled state.[7]

It is important that the photon emission is coherent. This is possible for resonant excitation as described. The experiment of Ref. 13 showed exciton dephasing times longer than 30 ns in InAs quantum dots. For a realistic radiative lifetime of 300 ps this would imply dephasing related errors below the 1% level. Resonant excitation requires separating the pump light from the photons that one wants to detect. This can be done temporally using a fast electro-optic switch. There are currently available Pockels cells with switching times shorter than 100 ps, which would already be enough to detect most of the desired photons.

A deviation from the conditions $E^A_{\sigma_+}=E^B_{\sigma_+}$ and $E^A_{\sigma_-}=E^B_{\sigma_-}$ by an amount δE will lead to an error in the Bell measurement due to imperfect wavefunction overlap of $(\delta E)^2/\gamma^2$, where γ is the inverse of the radiative lifetime. For a lifetime of 300 ps, one needs a precision of 0.2 μeV for an error of 1%. To achieve both conditions, one must be able to tune both the trion energy in zero field E_T, which can be done by varying the temperature,[14] and the Zeeman energy E_Z, which can be done by varying the magnetic field. The required precision of control can be estimated to be of order 5 mK for the temperature and of order 1 mT for the magnetic field. These values are realistic with present technology.

For a combined collection and detection efficiency $\eta=0.25$ for each photon, the proposed scheme allows to entangle two spins separated by 20 km in 8 ms, which is the same time as is obtained for the scheme of Ref. 5 with the same η and an emission probability of 8%. Note that the latter probability must be kept relatively small for the protocol of Ref. 5 to avoid errors due to the emission of two photons.

The superposition $\frac{1}{\sqrt{2}}(|1/2\rangle+|-1/2\rangle)$ required for entanglement creation can be realized via Raman transitions exploiting the fact that there are excited trion states that have significant dipole moments with both electronic ground states.[15,16] For our chosen field configuration, one of the two laser beams must propagate in a direction orthogonal to the growth axis. A detailed scheme for realizing arbitrary one-qubit operations via Raman transitions is described in Ref. 17. The most important error mechanism is the decoherence of the excited trion state. The related error can be estimated[18] to be below 10^{-3} for a decoherence rate $\gamma=3\times10^{10}$/s as in Ref. 19 and a realistic detuning of order 30 meV. Coherent manipulation of single spins in quantum dots via Raman transitions has recently been demonstrated experimentally.[20]

Qubit measurements can be realized via cycling fluorescence as proposed in Ref. 21. If σ_+ radiation is applied in resonance with the $|1/2\rangle\rightarrow|3/2\rangle_T$ transition, and the electron is originally in the $|1/2\rangle$ state, then the system will cycle between the $|1/2\rangle$ and $|3/2\rangle_T$ states emitting photons, whereas no photons will be emitted if the electron is originally in state $|-1/2\rangle$. The occurrence of "forbidden" transitions from $|3/2\rangle_T$ to $|-1/2\rangle$ limits the number of cycles that can be used for detection. However, the forbidden transitions are strongly suppressed in quantum dots with high cylindrical symmetry. In our numerical calculations on cylindrical quantum dots in a nanowire structure, cf. below, we found probabilities for the forbidden transition at the level of 10^{-3} per cycle, which allows of order 10^3 cycles. For experimental results showing precise optical selection rules in self-assembled quantum dots see Ref. 22. In practice, a mean number of 20 detected photons in combination with a threshold of 10 counts for a positive detection of the bright state will ensure that measurement errors are below 0.5%.

We will now describe how to implement local nodes consisting of two or more interacting spins. We propose to realize local two-qubit gates based on spin-selective excitation combined with the dipole-dipole interaction between trions in neighboring quantum dots, using a fixed-detuning variation of the protocol of Ref. 15. One again applies σ_+ radiation close to resonance with the transition from $|1/2\rangle$ to $|3/2\rangle_T$. An excitation will thus only happen if the electron is in state $|1/2\rangle$. If a trion is excited in the neighboring dot as well, an additional phase is accumulated due to the dipole-dipole interaction. The two spins acquire this phase only if they are both in the state $|1/2\rangle$, which makes it possible to realize a controlled phase gate. A phase due to the dipole-dipole interaction between excitons in a pair of quantum dots has recently been observed.[23] To enhance the interaction, the trions can be made to have permanent dipoles by applying an electric field orthogonal to the growth direction. For example, for two stacked flat quantum dots whose centers are separated by 10 nm, an electron-hole separation of 5 nm gives a dipole-dipole interaction energy E_{dd} of order 5 meV. There are different techniques for fabricating stacked quantum dots that are sufficiently close together. One promising approach is the use of heterostructured semiconductor nanowires as in Refs. 9.

The gate operation is performed adiabatically, i.e., the exciting laser is slightly detuned from the trion resonance. An important source of error for the two-qubit gates is spontaneous emission of photons from the trion state, the probability of which is $\Gamma\int dtP_T(t)$, where Γ is the spontaneous decay rate and $P_T(t)$ is the population in the trion state. For example, choosing a laser Rabi frequency $\Omega(t)=\Omega_0 e^{-t^2/\tau^2}$ with $\Omega_0=1.0\times10^{12}$/s, $\tau=11$ ps and a detuning $\Delta=0.75\times10^{12}$/s gives a controlled phase of π. For these values $\int dtP_T(t)$ is equal to 3.4 ps, which would give a 1.1% error for a trion

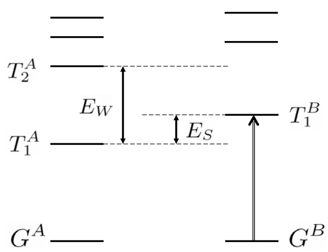

FIG. 2. Requirements imposed by spectral addressing. G is the quantum dot ground state, T_1, T_2, etc., are the trion states. Zeeman sublevels are not shown, i.e., G corresponds to the states $|\pm 1/2\rangle$ and T_1 to $|\pm 3/2\rangle_T$. Suppose that A is the dot with the lowest energy for T_1, and B another dot in the same node. When exciting the $G^B \rightarrow T_1^B$ transition, one must avoid exciting $G^A \rightarrow T_2^A$. This defines an energy window E_W in which T_1^B must lie. On the other hand, T_1^B must be larger than T_1^A by at least E_S in order to avoid exciting T_1^A while emitting a phonon.

lifetime of 300 ps as considered above. This error is reduced to 0.34% for a lifetime of 1 ns, for which the coherence and control requirements discussed above still appear realistic.

For the dipole-dipole interaction to be effective, the dots must be very close together. Individual dots must be addressed spectrally. This is possible exploiting the fact that the trion energies vary with the dimensions of the dot. The number of qubits per node is restricted by the existence of excited trion states, which imposes an energy window E_W for the trion energies $E^K_{\sigma_\pm}$ that can be used for addressing the qubits in a given node, and by the interaction with phonons, which requires a minimum energetic separation E_S between dots, cf. Fig. 2.

A typical quantum dot has several excited electron and hole states. While the electronic states are typically quite well described by the effective mass approximation for the electron (particle in a box), this is not the case for the hole states. In order to be able to make quantitative estimates, we have performed numerical calculations of the electronic properties for quantum dot structures in a model system. We have studied quantum dots of various dimensions that are constituted by layers of GaAs in an $Al_{0.4}Ga_{0.6}As$ nanowire with circular cross section. In order to simplify the discussion, we will focus on a particular example, namely a dot with 16 nm diameter and 4 nm thickness that is completely embedded in AlGaAs. Such a structure can be fabricated by performing the radial overgrowth[24] of an AlGaAs shell layer over a GaAs/AlGaAs axial nanowire heterostructure.[9] The one-particle states of the nanowires were computed in a tight-binding framework,[25] using the $sp^3d^5s^*$ model of Ref. 26 The lowest-lying electron and hole wave functions of the $\simeq$750 000 atoms supercell were computed with a Jacobi-Davidson algorithm as described in Ref. 25. A transverse electric field of 5 mV/nm is applied to introduce an electron-hole separation of 5 nm. The first four excited hole states for the described dot lie 15, 24, 26, and 30 meV above the hole ground state (not counting Zeeman sublevels). The first excited electron state is 48 meV above the electron ground state. The first excited trion state therefore consists of the electron in its ground state and the hole in its first excited state. In the presence of the electric field, the strength of this transition is about 1/4 of the lowest energy trion transitions, i.e., it is far from negligible. This implies that the energies of the (lowest-lying) trions for all dots in a node should lie in a window E_W of order 15 meV, cf. Fig. 2.

Light that is in resonance with T_1^B can excite T_1^A while emitting an acoustic phonon, cf. Fig. 2. Following Refs. 27 and 28 one can show that the rate for this process is given by $\gamma(\Delta,t)=2\pi J(\Delta)\Omega^2(t)/\Delta^2$, where Δ is the detuning, $\Omega(t)$ is the Rabi frequency of the laser and the function $J(\Delta) = \frac{\Delta^3}{16\pi^3\rho c^5}\int d^2\mathbf{n}|D(\Delta\mathbf{n}/c)|^2$. Here ρ is the density, c the sound velocity, the integral is over the surface of the unit sphere, and $D(\mathbf{k})=\int d\mathbf{r}[D_v|\psi_v(\mathbf{r})|^2-D_c|\psi_c(\mathbf{r})|^2]\exp(-i\mathbf{k}\cdot\mathbf{r})$, where D_c and D_v are the deformation potential constants for the dot material.[27] The wave functions ψ_v and ψ_c are those of the hole and electron ground states, respectively. The wave functions are obtained by the tight-binding calculations described above. For the above choice of $\Omega(t)$, one finds that a separation of E_S=7.5 meV (corresponding to the center of the energy window, since E_W=15 meV for our example) reduces the error due to phonon emission to 0.14%. Together with the error due to spontaneous emission of 0.34% predicted above, this means that it is possible to realize nodes containing two qubits such that the total error for two-qubit gates is of order 0.5%. Three qubits per node are possible if one tolerates a total error of order 2%. We have focused on the two-qubit example in order to facilitate comparison with Ref. 5, which shows that a quantum repeater protocol with two-qubit nodes and local errors for two-qubit gates and measurements (cf. above) of 0.5% is rather efficient. For example, it would allow to establish an entangled pair over 1000 km in a few seconds, with neighboring nodes separated by 20 km, as in our above discussion. We have thus shown that our proposed scheme is capable of the same performance, without the requirement of phase stability for the optical fiber links.

We studied the model system of GaAs in AlGaAs because all the relevant parameters are sufficiently well known to make quantitative predictions. However, equivalent results are to be expected for appropriate II-VI systems,[29] which have the advantage of allowing the elimination of nuclear spins, as mentioned above. For example, for ZnSe the effective hole masses are about 50% larger than for GaAs, which might lead to a proportionately smaller energy window. However, the deformation potential constants are predicted to be significantly smaller than for GaAs, which would lead to a smaller required separation for the same dot dimensions. The chosen dot dimensions are the result of an (informal) optimization. Reducing the dot dimensions, e.g., increases the level separation (and thus E_W), but it also makes the function $J(\Delta)$ wider, and thus increases E_S.

The spectral addressing requirements for the qubit measurements are less severe than for the two-qubit gates because the necessary light intensities are smaller. For the one-qubit gates, since the Raman lasers are far detuned from the trion energies, the trion resonances cannot be used to address individual dots. However, since for the Raman process the difference in laser frequencies must be equal to the energy difference between the two qubit states, one can use the variation in Zeeman energies between individual dots for addressing. The electron g factors vary with the size of the quantum dots. Published results on self-assembled dots in III-V and II-VI systems[30–32] suggest that it is quite feasible to achieve a variation in Zeeman energy of order 1 μeV in a 1 T magnetic field for dots whose trion energies differ by 7.5 meV. This is consistent with gate times for the one-qubit gates below 10 ns, with negligible addressing errors.

We have shown how to create entanglement between remote quantum dot spins by first entangling the spins with photons emitted by the dots, and then detecting the two photons in the Bell basis. We have demonstrated that it is possible to realize local nodes consisting of two or more quantum dots such that nearest neighbors are coupled by dipole-dipole interactions between excitons. Based on a detailed study of expected errors and a comparison with the results of Ref. 5, we have shown that our proposed protocol should allow the realization of efficient quantum repeaters without requiring interferometric stability.

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